A Note on the slope-shift anticorrelation in the neutron star kHz QPOs data

Marek A. Abramowicz[1,6], Didier Barret[2],
M. Bursa[3], Jiří Horák[3], Włodek Kluźniak[4]
,Paola Rebusco[5], Gabriel Török[6]

[1] *Department of Physics, Göteborg University,
S 412 96 Göteborg, Sweden*
[2] *Centre d'Etude Spatiale des Rayonnements,
CNRS/UPS, 9 Avenue du Colonel Roche, 31028 Tolouse Cedex 04, France*
[3] *Astronomical Institute, Academy of Sciences,
Boční II 1401, 141 31 Praha 4, Czech Republic*
[4] *Zielona Góra University,
Lubuska 2, 65-265 Zielona Góra, Poland*
[5] *Max-Planck-Institute für Astrophysik,
D-85741 Garching, Germany*
[6] *Institute of Physics, Silesian University in Opava,
Bezručovo nám. 13, 746 01 Opava, Czech Republic*

ABSTRACT
Observations show that the upper ν_U and lower ν_L of the "twin peak" high frequency QPOs in neutron star sources vary along their "Bursa lines", $\nu_U = A\,\nu_L + B$, and that the ratios ν_U/ν_L cluster near 3/2. This behaviour is predicted by the non-linear resonance model for QPOs proposed by Kluźniak & Abramowicz. In this Note, we futher explore our recent finding that the coefficients A, B of the Bursa lines for individual sources are anticorrelated. In the (A, B) plane, they occupy a narrow triangle along the line $A = A_0 - B/\nu_0$, with $A_0 \approx 1.5$, and $\nu_0 \approx 600$ Hz. We show that this observational property of QPOs also follows from the resonance model.

Keywords: LMXRB – neutron stars – X-ray variability – observations – theory

1 THE BURSA LINE

The Fourier power spectra of X-ray variability from Galactic neutron star and black hole sources often reveal twin peaks corresponding to two physically connected frequencies — upper ν_U and lower ν_L. These frequencies are very high, from hundreds to thousands of Hertz — i.e. in the range of ISCO frequencies. Thus, most likely, the oscillations, which produce them, occur very near the central compact source, in the super strong Einstein gravity.

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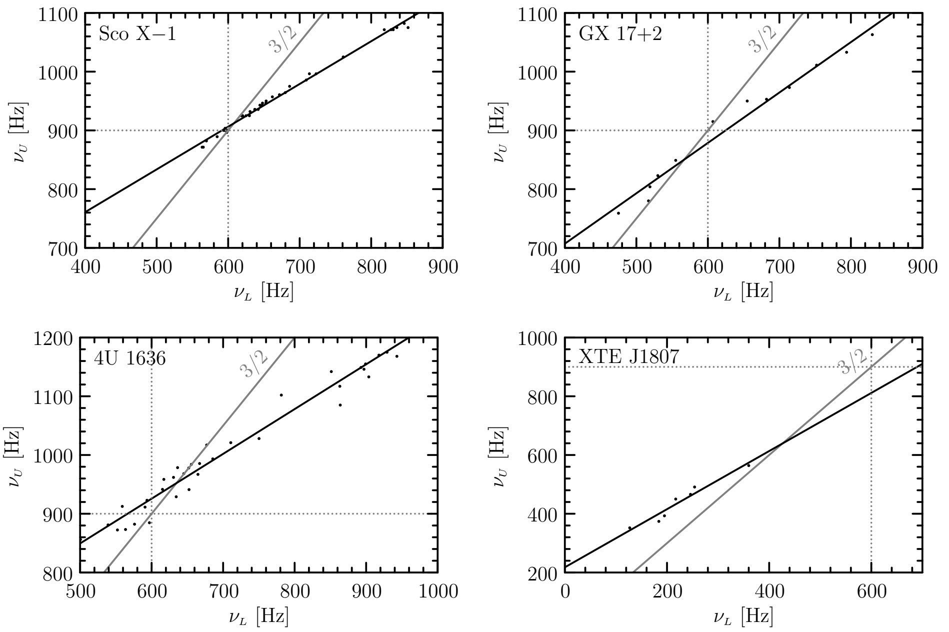

Figure 1. The Bursa lines for individual sources. The two Z-sources (Sco X-1, GX 17+2), one atoll source (4U 1636) and the millisecond pulsar XTE J1807 are shown. The slopes and shifts A and B of their best linear fits are listed in the Table 1. Note that all the Bursa lines come close to the point [600 Hz, 900 Hz] shown as intersection of two dotted lines.

In the black hole sources, ν_U and ν_L appear to be fairly fixed, and in addition to have a well defined rational ratio $\nu_U/\nu_L = 3/2$, as first noticed by [Abramowicz & Kluźniak(2001)]. In the neutron star sources ν_U and ν_L vary by hundreds of Hertz, along “Bursa lines” (see e.g. Abramowicz 2005, also Figure 1),

$$\nu_U = A\,\nu_L + B. \tag{1}$$

It was demonstrated by [Abramowicz et al.(2003b)], [Rebusco (2004)], and [Horák (2004)] that variations of ν_U and ν_L along the Bursa line (1) follows from the Kluźniak & Abramowicz non-linear resonance model for QPOs, as we shortly recall in Section 3.

The four examples shown in Figure 1 illustrate another important general observational feature of the neutron star QPOs. The sectors occupied by data points on the individual Bursa lines always cross the “3/2” straight line. This obviously underlines the relevance of the 3/2 ratio not only for the black hole QPOs but for the neutron star QPOs as well, as first noticed and stressed by [Abramowicz et al.(2003a)]. They demonstrated, using a relatively small data sample available to them at that time, that for Sco X-1 the observed ν_U/ν_L ratios cluster very near, but not exactly at, the 3/2 value. This claim was initially challenged by [Belloni et al.(2004)], but a more detailed analysis by [Belloni et al.(2005)], with a larger sam-

Table 1. Best linear fits and their errors for the frequency–frequency correlation for several atol and Z sources (A and Z, respectivelly) and for the millisecond pulsar (P). The references: 1–6) [Abramowicz et al. (2005b)], 7) [Boirin et al. (2000)], 8) [Linares et al. (2005)], 9) [Homan et al. (2002)], 10) [Jonker et al. (2000)], 11) [Jonker et al. (2002)], 12) [Belloni et al.(2005)].

Source		Type	A	ΔA	B [Hz]	ΔB [Hz]
1)	4U 0614	A	1.02	0.03	302	23
2)	4U 1728	A	1.00	0.05	352.4	39.6
3)	4U 1820	A	0.93	0.05	322.8	35.8
4)	4U 1608	A	0.75	0.003	459.4	16.6
5)	4U 1636	A	0.72	0.01	503	8.1
6)	4U 1735	A	0.61	0.05	593	39
7)	4U 1915	A	1.13	0.03	266	16
8)	XTE J1807	P	1.11	0.11	181	26
9)	GX 17+2	Z	0.86	0.04	364	28
10)	GX 34+0	Z	0.84	0.07	391	26
11)	GX 5-1	Z	0.83	0.04	386	14
12)	Sco X-1	Z	0.786	0.002	432.5	1.5

ple and for more neutron star sources, confirmed that the clustering near 3/2 (and less often at other rational ratios as first suggested by [Abramowicz et al.(2003b)] see their Fig. 2) is indeed real.

2 THE SLOPE-SHIFT ANTICORRELATION

Recently, [Abramowicz et al.(2005)] discovered another interesting feature of the frequency-frequency plots. They noticed that the *individual* crossing points between the *individual* Bursa lines and the "3/2" line, all fall rather close to a single "magic" point [600 Hz, 900 Hz] which means that,

$$\nu_{\rm U} - 900\,{\rm Hz} \approx A\,(\nu_{\rm L} - 600\,{\rm Hz}) \tag{2}$$

They discussed this surprising magic by showing an obvious consequence of (2) — the coefficients $A =$ slope, $B =$ shift of the individual Bursa lines (1) consistent with (2) must be anticorrelated,

$$A \approx \frac{900\,{\rm Hz}}{600\,{\rm Hz}} - \frac{B}{600\,{\rm Hz}}. \tag{3}$$

Abramowicz & al. (2005) had analysed six atol sources and for each source, they found the slope A and the shift B directly from the best linear fit, with no preassumptions, in particular no preassumption about the magic point. Here, we

Table 2. Anticorrelation between the shift and the slope for 12 sources. The first two lines compare best linear fits $A(B)$ separately for coherent set of sources (Table 1: 1–6) analyzed in [Abramowicz et al. (2005b)] and for six others sources (Table 1: 7–12) examined by different groups. The second pair of lines includes all sources and compare the $A(B)$ best linear fit vs. fit with intercept 1.5 shown in Fig. 2. Note that this two fits have almost the same quality (see also Fig. 3 for conjuctions).

Sources	Best fit $A(B)$	$\frac{\chi^2}{\mathrm{dof}}$
1–6	$A = 1.45(\pm 0.09)B - 0.0015\,(\pm 0.0002)$	0.48
7–12	$A = 1.58(\pm 0.09)B - 0.0018(\pm 0.0002)$	0.60
1–12	$A = 1.44(\pm 0.06)B - 0.0015(\pm 0.0001)$	1.62
1–12	$A = 1.5B - 0.0016$	1.84

repeat their analysis [1] and adding six more sources, including Z-sources and a milisecond pulsar. All twelve sources are listed in Table 1, where we also give the resulting A, B. Table 2 compare the relation between A, B obtained by least squares method for coherent set of six sources analyzed in [Abramowicz et al. (2005b)] and for six others sources analyzed by different groups, as well as the best linear fit for all 12 sources. This comparison shows that these 12 sources are well consistent with relation

$$A = 1.5 - 0.0016B, \tag{4}$$

which is also illustrated in Fig. 2.

3 THEORETICAL EXPLANATION OF THE LINEAR FREQUENCY CORRELATION AND THE SLOPE-SHIFT ANTICORRELATION

The frequency and the amplitude of a non-linear oscillator are not independent. In the lowest order with respect to the small amplitude α, the actual (observed) frequency ν differs from the eigenfrequency ν^0 of the oscillator by a correction $\Delta\nu$ proportional to the squared dimensionless amplitude, $\nu - \nu^0 = \Delta\nu \sim \nu^0\, a^2$. Consider a very general system that has two coupled oscillation modes, whose eigenfrequencies are ν^0_{L} and ν^0_{U}. The frequencies of non-linear oscillations may be written in the form

$$\nu_{\mathrm{L}} = \nu^0_{\mathrm{L}} + \Delta\nu_{\mathrm{L}}\,, \quad \nu_{\mathrm{U}} = \nu^0_{\mathrm{U}} + \Delta\nu_{\mathrm{U}}\,, \tag{5}$$

$$\Delta\nu_{\mathrm{L}} = \kappa_{\mathrm{L}} a^2_{\mathrm{L}} + \kappa_{\mathrm{U}} a^2_{\mathrm{U}}\,, \quad \Delta\nu_{\mathrm{U}} = \lambda_{\mathrm{L}} a^2_{\mathrm{L}} + \lambda_{\mathrm{U}} a^2_{\mathrm{U}}\,, \tag{6}$$

[1] For description of details of the data analysis (some of them non trivial) see [Abramowicz et al. (2005b)].

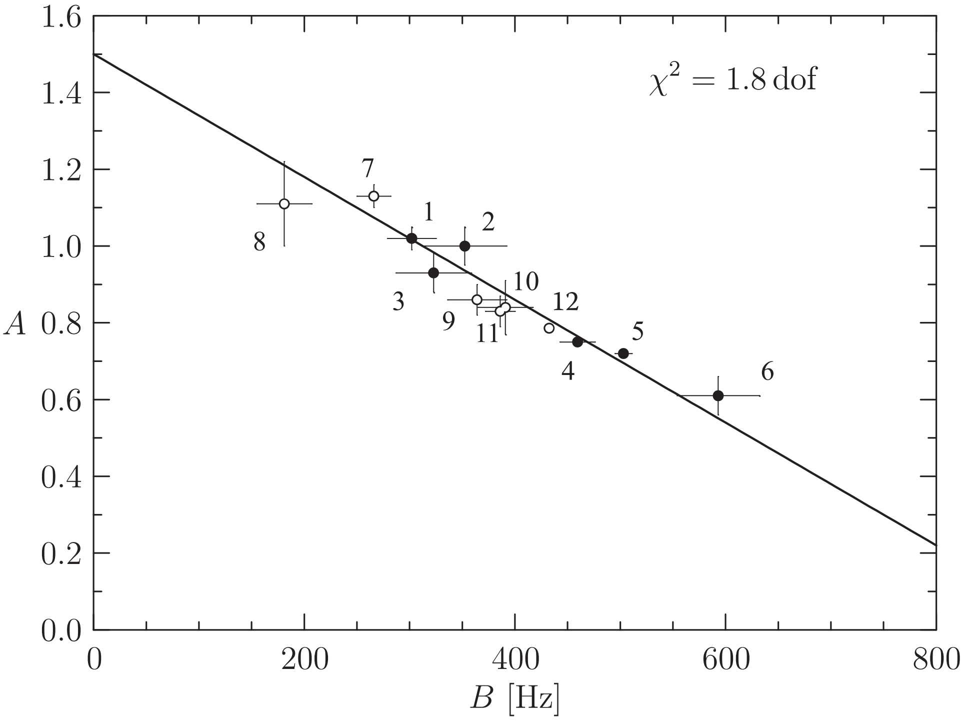

Figure 2. The anticorrelation between slopes and shifts. The points correspond to different sources listed in the Table 1; sources 1–6 (7–12) are denoted by filled (open) circles. We also show the best fit for the line going through the point $[0, 1.5]$ and the corresponding value of χ^2.

where κ_{L}, κ_{U}, λ_{L} and λ_{U} are constants depending on non-linearities in the system and a_{L} and a_{U} are amplitudes of the oscillators.

It is natural to suppose that due to an interplay between the resonance excitation mechanism and the dissipation of the energy in the system, the two amplitudes a_{L} and a_{U} are correlated, i.e. one may consider the amplitudes as functions of a single parameter s,

$$a_{\mathrm{L}} = a_{\mathrm{L}}(s)\,, \quad a_{\mathrm{U}} = a_{\mathrm{U}}(s)\,. \tag{7}$$

Expanding with respect to s, we obtain from equation (5)

$$\nu_{\mathrm{L}} = \tilde{\nu}_{\mathrm{L}} + s\,F, \quad \tilde{\nu}_{\mathrm{L}} = \nu_{\mathrm{L}}^{0}\left(1+\tilde{\alpha}_{\mathrm{L}}^{2}\right), \quad F = f_0 + f_1 s + ..., \quad \tilde{\alpha}_{\mathrm{L}}^{2} \ll 1, \tag{8}$$

$$\nu_{\mathrm{U}} = \tilde{\nu}_{\mathrm{U}} + s\,G, \quad \tilde{\nu}_{\mathrm{U}} = \nu_{\mathrm{U}}^{0}\left(1+\tilde{\alpha}_{\mathrm{U}}^{2}\right), \quad G = g_0 + g_1 s + ..., \quad \tilde{\alpha}_{\mathrm{G}}^{2} \ll 1, \tag{9}$$

Isolating the parameter s from the two last equations, we get the Bursa line, i.e. a linear correlation between the observed frequencies $\nu_{\mathrm{U}} = A\nu_{\mathrm{L}} + B$, with the slope A and the shift B given by,

$$A = \frac{\tilde{\nu}_{\mathrm{U}}}{\tilde{\nu}_{\mathrm{L}}}\,X\,, \quad B = \tilde{\nu}_{\mathrm{U}}(1-X), \quad X \equiv \frac{G}{F}. \tag{10}$$

Any particular value of X leads to particular values of the slope and the shift. By solving equations (10) for the parameter X, one gets

$$A = \frac{\tilde{\nu}_{\mathrm{U}}}{\tilde{\nu}_{\mathrm{L}}} - \frac{1}{\tilde{\nu}_{\mathrm{L}}}\,B \; = A_0 - \frac{1}{\tilde{\nu}_{\mathrm{L}}}\,B\,. \tag{11}$$

Note that *for a given type of resonance* $A_0 = \mathrm{const} + \mathcal{O}(a^2)$, as it depends only on the ratio of the amplitude corrected eigenfrequencies. Of course, $\tilde{\nu}_{\mathrm{L}} \neq \mathrm{const}$ even for a given resonance, as the eigenfrequencies themselves may differ from a system to another system.

Therefore, equation (11) predicts that the slope A and the shift B are anticorrelated. For a given type of resonance, the individual pairs A, B should locate on lines inside a triangle with a quite well determined vertex at [0, A_0], and with the size of its base proportional to the scatter in $\tilde{\nu}_{\mathrm{L}}$. In particular, for the 3:2 resonance the vertex should be very close to the point [0, 1.5], which seems to be indeed the case, as Figure 3 shows.

4 MASSES AND ROTATION RATES OF NEUTRON STARS

The neutron star masses M must enter the discussion because of the $1/M$ scalling of QPO frequencies predicted by the Kluźniak & Abramowicz resonace model, and by *all models* that assume strong gravity origin of QPOs. Indeed, in a strong gravity, a typical size is of the order of the graviatational radius, $r_{\mathrm{G}} \sim M$, a typical velocity of the order of the light speed c and therefore the typical frequency is $\nu \sim c/r_{\mathrm{G}} \sim 1/M$. The frequency scales inversely with the mass if the mass is the main difference between neutron stars.

Thus, the $\tilde{\nu}_{\mathrm{L}}$ frequency of equation (11) should roughly scale inversly with the mass of individual neutron star corresponding to an individual A, B point. If all neutron star masses were equal, all the $\tilde{\nu}_{\mathrm{L}}$ frequencies would be equal as well, and the magic point would be a single point. However, the masses of neutron stars are not equal and therefore the magic point is really a region with the size proportional to the range of masses of the neutron star sources involved — the smaller the range, the more point-like the region is.

It is easy to see that the slope-shift anticorrelation line is steeper or softer for more massive or less massive sources, respectively. The shaded regions in Figure 3 correspond to mass ranges given in each panel.

The issue is more complicated because the $1/M$ scalling is not exact – it is also affected by rotation and internal structure of neutron stars. In this sense, the anticorrelation has a great potential in providing observational contrains for masses, rotation rates, and multipole momenta for neutron stars.

5 BLACK HOLE SOURCES

In principle, our theory can be applied to the case of black hole QPOs as well. The apparently steady frequencies reported in these systems can be attributed to smaller

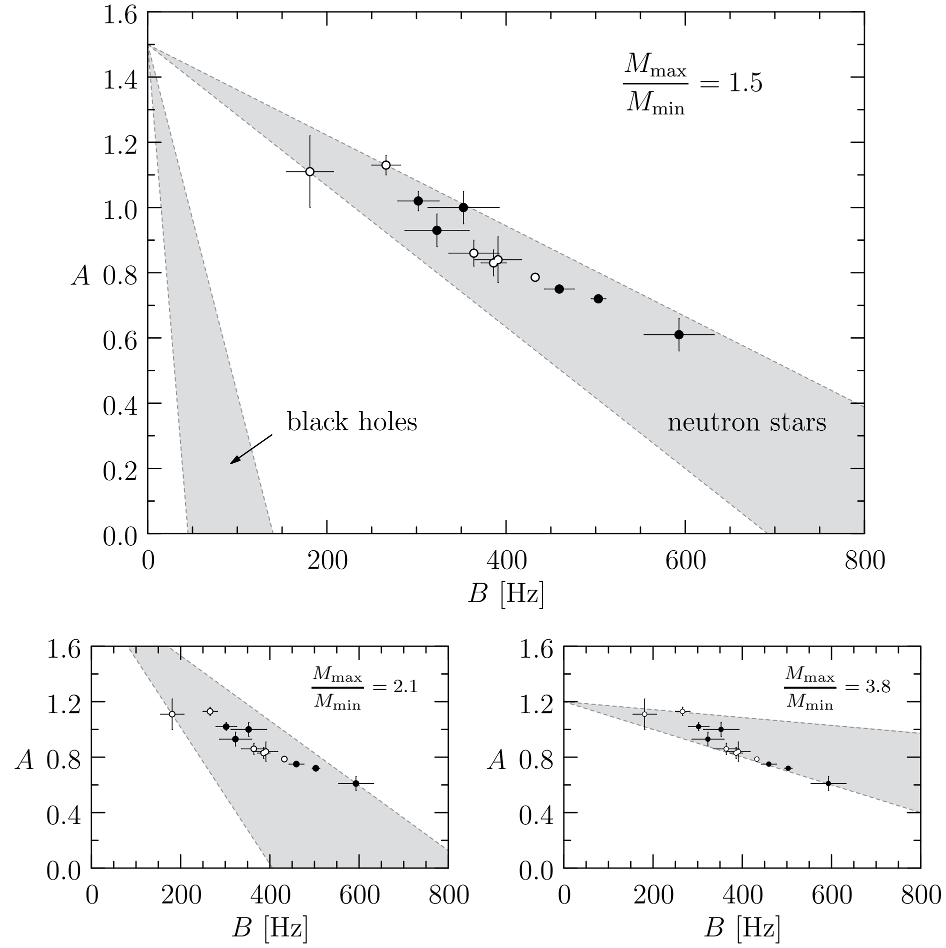

Figure 3. The $1/M$ scaling and the slope-shift anticorrelations. The characteristic frequencies scale inversely with the neutron stars masses. On the other hand, the ratio $\tilde{\nu}_U/\tilde{\nu}_L$ should be close the particular value $A_0 = 3/2$ for the resonance. The shaded regions give us the spread of the neutron star masses and expected region of black hole sources (see the text). We consider three cases: $A_0 = 3/2$ (top), 2.0 (bottom-left) and 1.2 (bottom-right). Note the rather unrealistic ratio between expected maximal and minimal neutron star masses in the sample for the values $A_0 \lesssim 1.2$.

eigenfrequencies and amplitudes of oscillations by a simple argument. Suppose that the excitation mechanisms are similar in both black holes and neutron stars. Therefore the dimensionless coefficients λ_L, λ_U, κ_L and κ_U in equations (6) are of the same order. The frequencies observed from black holes differ from typical ones of neutron stars (i.e. frequencies in the magic point) by a factor of about two[2]. The same can be applied to the eigenfrequencies. If the amplitudes of black hole QPOs were smaller than that of the neutron stars by a factor of ~ 5, the range of observed frequencies would be shorter by a factor of ~ 50 with respect to the range of the neutron stars.

The reported rms amplitudes of black hole QPOs are only a few percent contrary to neutron star QPO rms ampliudes that usually exceed ten percent. The question is, how are the observed rms amlitudes of OPOs in X-ray flux related to the intrinsic amplitudes of oscillations. This is obviously connected to the modulation mechanism that may be different in the two classes of objects. If, similarly, the instrinsic amplitudes differ by such large factor, it should not be surprising that the QPO frequencies appear stable, since the predicted range of their variations correspond to a few Hertz. Such small variations can not be ruled out, more to the contrary, [Miller et al. (2001)] reported systematic motion of the upper high-frequency peak towards lower freqeuncies in the source XTE J1550-564. At this general level, the black hole QPOs can be understood as rescaled version of the neutron star QPOs and the corresponding region in the slope-shift plot is also denoted in Fig 3.

6 DISCUSSION AND CONCLUSIONS

Both, the Bursa lines and the slope-shift anticorrelation, are generic, and very general, consequences of non-linearity and coupling of oscillation modes, as explained above. The fact that observations of the neutron stars QPOs show both the Bursa line and the slope-shift anticorrelation, is a strong clue that the QPOs are due to non-linear, coupled oscillations. Another observational clue is that the eigenfrequency ratio is close to the rational ratio, $\nu^0_{\rm U}/\nu^0_{\rm L} = 3/2$, which suggests a 3:2 resonance. At the moment, one cannot firmly conclude anything more than that from observational data.

We finish this note by putting two open questions related to this issue:
(i) What physical processes and parameters determine the value of the X-parameter in equations (10)?
(ii) What physical parameters give the range of observed QPO frequencies?

ACKNOWLEDGEMENTS

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[2] The dirrect application of the $1/M$ scaling rule gives a factor of ~ 5. However, large differences in spin between black holes and neutron stars break such simple scaling between the two classes of objects (so that e.g. the 5 times hevier black hole may have QPO frequencies only 2 times lower than a typical neutron star).

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