

# A possible mechanism for QPOs modulation in neutron star sources

J. HORÁK

Astronomical Institute, The Czech Academy of Sciences, Boční II, 140 31 Prague, Czech Republic

Received 8 September 2005; accepted 22 September 2005; published online 20 October 2005

**Abstract.** It was pointed out by Paczyński (1987) that the X-ray luminosity of accreting neutron stars is very sensitive to the physical properties of the accretion flow close to the innermost stable circular orbit. The X-ray radiation is dominated by that emitted in the boundary layer, where accreted matter hits a star surface. The X-ray luminosity of the boundary layer is proportional to the local accretion rate. In this note, we estimate a local accretion rate variations from the disk that undergoes non-stationary axisymmetric perturbations. The perturbations are given by the poloidal-velocity potential. We obtain a simple formula describing the modulation of the accretion rate for the particular case of global vertical disk oscillations that have been recently studied by Abramowicz et al. (2005).

**Key words:** stars: neutron – quasi-periodic oscillations – accretion – gravitation

©2005 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim

## 1. Introduction

The quasi-periodic oscillations (QPOs) appear in the light curves of more than 20 bright low-mass X-ray binaries (LMXBs) with accreting neutron stars (van der Klis 2000). Much attention is attracted to the kilohertz QPOs because their frequencies are comparable with orbital frequencies in the innermost parts of the accretion disks. The orbital frequency of a particle orbiting the neutron star of mass  $M$  at the innermost stable circular orbit (ISCO) is  $\nu_{\text{ISCO}} = 1580(1 + 0.75j) \text{ Hz} \times 1.4 M_{\odot}/M$  (Kluźniak et al. 1990).

Many models have been proposed to explain the excitation mechanism of QPOs and subsequent modulation of the X-ray signal (see van der Klis 2000 and McClintock & Remillard 2003 for a detailed discussion of observations and models). Recently, it has been suggested that the high frequency QPOs arise from a resonance between two oscillation modes of the innermost part of the accretion disk (?). In the case of the neutron-star sources, the modulation of the X-ray radiation may originate in the modulation of the local accretion rate (Kluźniak & Abramowicz 2004).

In LMXBs that are not pulsars, the magnetic field of the neutron star is sufficiently weak, allowing the accretion disk to extend down to ISCO. The strongest X-ray radiation then originates in the boundary layer, where accreted material hits the star surface. Depending on the star radius  $R_*$ , the amount of energy released in the boundary layer exceeds that radiated

by the whole disk. It gives about 69% of the total luminosity if  $R_* = 3R_S$ , or even 86% if  $R_* = 1.5R_S$ .

In this context, Paczyński (1987) pointed out that a variability of X-ray luminosity of accreting neutron stars may be governed by physical properties of the accretion flow close to ISCO. In Einstein gravity theory, the inner edge of the pressure supported thick accretion disks is slightly below ISCO (Abramowicz 1985). The material is accreted from the disk through a narrow potential nozzle onto the neutron star. Obviously, if the innermost part of the disk is not stationary but is a subject to some oscillations then the fine structure of the flow at the inner disk edge is significantly changed. This strongly affects the accretion rate through the nozzle and the resulting X-ray luminosity of the boundary layer. This scenario is in agreement with the recent observations of Gilfanov et al. (2003) and more recently Revnivtsev & Gilfanov (2005) that strongly point to the fact that neutron-star QPOs are modulated in the boundary layer.

In Sects. 2 and 3 we briefly summarize equations important for the disk structure close to the ISCO and reproduce the calculations of the accretion rate through the inner edge of the stationary disk. Then in Sect. 4 we calculate the accretion rate from the disk that is subject to nonstationary axisymmetric perturbations. We derive a simple formula for the accretion rate modulation of a vertically oscillating disk.

## 2. Disk structure close to ISCO

We consider an axisymmetric thick disk made of a perfect fluid surrounding a neutron star of mass  $M$ . The dynamics of the fluid is governed by Euler equation, poloidal component of which takes the form

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} - \frac{\ell^2}{r^2} \mathbf{e}_r + \frac{\nabla p}{\rho} - \nabla \Phi = 0, \quad (1)$$

where the bold-face letters refer to the poloidal part of the vectors,  $\mathbf{a} \equiv (a^r, a^z)$ ,  $\Phi$  is a gravitational potential,  $r$  denotes radial coordinate (we employ the cylindrical coordinates  $\{r, \phi, z\}$ , with the origin coinciding with the center of the star) and  $p$ ,  $\rho$  and  $\ell$  are the pressure, density and the angular momentum of the orbiting flow respectively (in general all dependent on  $r$  and  $z$ ). The azimuthal component of the Euler equation gives conservation of angular momentum,

$$\frac{\partial \ell}{\partial t} + \mathbf{v} \cdot \nabla \ell = 0. \quad (2)$$

We assume that the angular momentum is constant in the whole volume of the disk,  $\ell(r, z) = \ell_0$ , and that the fluid obeys the polytropic equation of state,  $P = K\rho^{1+1/n}$ , where  $K$  and  $n$  are polytropic constant and polytropic index, respectively. In addition, we assume that the poloidal velocity  $\mathbf{v} = (v^r, v^z)$  can be derived from the potential  $\chi$ . Hence, the Eq. (2) is satisfied automatically and the Eq. (1) can be further integrated to Bernoulli equation,

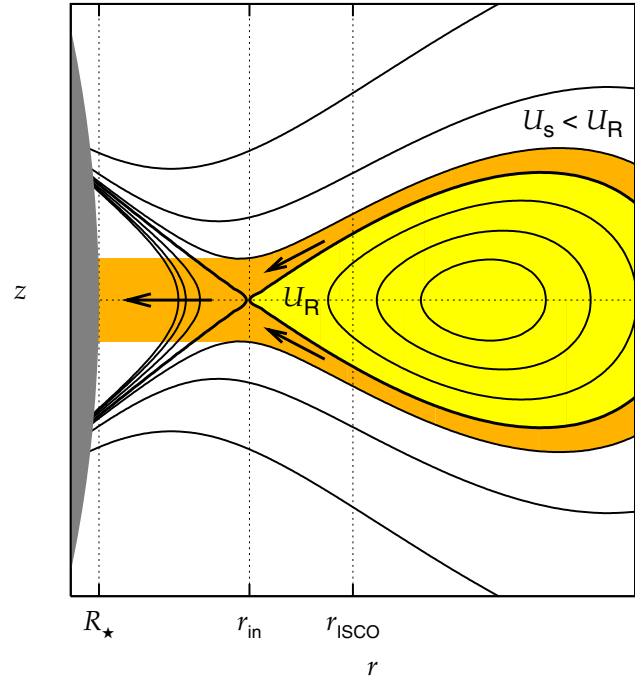
$$\frac{\partial \chi}{\partial t} + \frac{v^2}{2} + h + \mathcal{U} = \text{const} \equiv \mathcal{U}_s. \quad (3)$$

Here we introduced the poloidal-velocity potential by  $\mathbf{v} = \nabla \chi$ , the enthalpy of the fluid  $h \equiv \int_0^p dp/\rho = nK\rho^{1/n}$  and the effective potential  $\mathcal{U} = \Phi(r, z) + \ell_0^2/2r^2$ .

As a model of a strong gravitational field of the star, we use the pseudo-Newtonian potential  $\Phi(r) = -GM/(R - R_S)$ , where  $R \equiv \sqrt{r^2 + z^2}$  and  $R_S$  is Schwarzschild radius. It was introduced by Paczyński & Wiita (1980) and allows us to model general relativistic effects using Newtonian calculations with remarkable simplicity. Particularly, it gives a correct position of the marginally stable orbit at  $r = r_{\text{ISCO}} = 3R_S$  and well reproduces the Keplerian angular momentum of test particles orbiting the star,  $\ell_K = \sqrt{GM r^3/(r - R_S)}$ . The angular momentum is not a monotonic function of  $r$ , as it is in Newtonian gravity ( $R_S \rightarrow 0$ ). Instead, it has a minimum at the marginally stable orbit.

The structure of the stationary disk is shown in Fig. 1. The inner edge is at radius  $r = r_{\text{in}}$ , where the angular momentum of the flow equals the Keplerian value,  $\ell_0 = \ell(r_{\text{in}})$ . The Lagrange point is at coordinates  $[r_{\text{in}}, 0]$ . The equipotential surface corresponding to the value  $\mathcal{U}_R$  that crosses itself at the Lagrange point is called Roche lobe. The equilibrium configuration exists only if the surface of the torus is inside the Roche lobe, e.g. when  $\mathcal{U}_s \leq \mathcal{U}_R$  (Boyer 1965; Abramowicz et al. 1978).

Otherwise, the dynamical equilibrium is impossible and the overflowed matter will be accreted through the potential nozzle onto the star.



**Fig. 1.** Accretion from a thick stationary accretion disk. The position of the disk inner edge and the shape of the equipotentials are determined by the distribution of the fluid angular momentum. Here we consider the simplest case of the constant distribution,  $\ell(r, z) = \ell_0$ . The plot shows projections of the equipotential surfaces to the poloidal plane (solid lines) and the distribution of the fluid (shaded region). The matter that overflows the Roche lobe (the equipotential surface that crosses itself) is accreted onto the neutron star.

## 3. Stationary flow

The stationary accretion rate for Roche overflow was first calculated by Kozłowski et al. (1978), who used Einstein's theory. Here we closely follow the Newtonian calculations of Abramowicz (1985). We consider a small overflow, so that all quantities can be expanded to the second order in the vicinity of the Lagrange point  $L$ . Particularly, the vertical profile of enthalpy can be expressed as

$$h(r_{\text{in}}, z) = h^* - \frac{1}{2} \kappa^2 z^2, \quad \kappa^2 \equiv - \left( \frac{\partial^2 h}{\partial z^2} \right)_L, \quad (4)$$

where  $h^* \equiv h(r_{\text{in}}, 0)$  denotes a maximal value of the enthalpy on the cylinder  $r = r_{\text{in}}$ . The linear order does not contribute because the flow is symmetric with respect to the equatorial plane. The thickness of the inner edge is  $H = \sqrt{2h^*/\kappa}$ . Close to  $r = r_{\text{in}}$  the accretion flow becomes transonic. After Abramowicz (1985), we assume that the radial velocity of the flow equals to the local sound speed and that the vertical component of the velocity is negligible compared to the radial one. This significantly simplifies the solution because it allows us to express the poloidal velocity using the enthalpy,

$$\mathbf{v} = \sqrt{\frac{h}{n}} \mathbf{e}_r. \quad (5)$$

The local mass flux through the nozzle is  $\dot{m} = \rho v^r = \rho c_s = h^{n+1/2}/K^n (1+n)^n n^{1/2}$  and the integration over the

cylinder  $r = r_{\text{in}}$  gives the total mass flux in terms of the central enthalpy  $h^*$

$$\begin{aligned} \dot{M} &= \int_0^{2\pi} r_1 d\phi \int_{-H}^H \dot{m} dz \\ &= (2\pi)^{3/2} \frac{r_{\text{in}}}{n^{1/2}} \left[ \frac{1}{K(n+1)} \right]^n \frac{\Gamma(n+3/2)}{\Gamma(n+2)} \frac{(h^*)^{n+1}}{\kappa}, \end{aligned} \quad (6)$$

where  $\Gamma(x)$  is the Euler gamma function.

In the Bernoulli equation (3) we keep the term  $v^2/2$  and neglect only the time derivative because of stationarity of the flow. We obtain

$$\frac{v^2}{2} + h + \mathcal{U} = \left(1 + \frac{1}{2n}\right) h + \mathcal{U} = \mathcal{U}_S. \quad (7)$$

The parameter  $\kappa$  that determines the shape of the enthalpy profile can be expressed using a derivative of the effective potential. That introduces the vertical epicyclic frequency  $\omega_z$  to the problem. From Eq. (7) we obtain

$$\kappa^2 = \left(\frac{n}{n+1/2}\right) \omega_z^2, \quad \omega_z = \left(\frac{\partial^2 \mathcal{U}}{\partial z^2}\right)_L. \quad (8)$$

By substituting Eqs. (7) and (8) and introducing  $\Delta \mathcal{U} \equiv \mathcal{U}_0 - \mathcal{U}_S$  we finally recover the result obtained by Abramowicz (1985),

$$\dot{M} = A(n) \frac{r_{\text{in}}}{\omega_z} \Delta \mathcal{U}^{n+1}, \quad (9)$$

$$\begin{aligned} A(n) &\equiv (2\pi)^{3/2} \left[ \frac{1}{K(n+1)} \right]^n \left[ \frac{1}{n+1/2} \right]^{n+1/2} \times \\ &\times \frac{\Gamma(n+3/2)}{\Gamma(n+2)}. \end{aligned} \quad (10)$$

#### 4. A perturbed flow

Now, we suppose that the disk is disturbed and oscillates. In that case, the accretion flow will not be stationary anymore and in order to describe the flow we must use the Bernoulli Eq. (3) in the full form. The presence of the ‘non-stationary’ term  $\partial \chi / \partial t$  breaks however the correspondence between the enthalpy and the effective potential. The equipotential surfaces and the surfaces of constant enthalpy will not coincide anymore. If the oscillations are a small perturbation, we can expand the Bernoulli equation in the vicinity of the stationary flow considered above.

We suppose that the velocity potential can be expressed as

$$\chi(\mathbf{r}, t) = \chi_{(0)}(\mathbf{r}) + \epsilon \chi_{(1)}(\mathbf{r}, t), \quad (11)$$

where the subscript ‘(0)’ refers to the stationary flow and the dimensionless parameter  $\epsilon$  characterizes strength of the perturbation. We assume  $\epsilon \ll 1$ . Then, using the definition  $\mathbf{v}(\mathbf{r}, t) = \nabla \chi(\mathbf{r}, t)$  we find

$$\begin{aligned} v^2 &= v_{(0)}^2 + 2\epsilon \mathbf{v}_{(0)} \cdot \mathbf{v}_{(1)} + \epsilon^2 v_{(1)}^2 \\ &= c_s^2 + 2\epsilon c_s \frac{\partial \chi_{(1)}}{\partial r} + \epsilon^2 \left[ \left(\frac{\partial \chi_{(1)}}{\partial r}\right)^2 + \left(\frac{\partial \chi_{(1)}}{\partial z}\right)^2 \right]. \end{aligned} \quad (12)$$

The perturbation affects also the enthalpy. The new value can be approximated by an expansion via the parameter  $\epsilon$

$$h = h_{(0)} + \epsilon h_{(1)} + \epsilon^2 h_{(2)} + \mathcal{O}(\epsilon^3). \quad (13)$$

By substituting into the Bernoulli equation (3) and equating coefficients of same powers of  $\epsilon$ , we get

$$h_{(1)} = -\frac{\partial \chi_{(1)}}{\partial t} - \left(\frac{\bar{h}}{n}\right)^{1/2} \frac{\partial \chi_{(1)}}{\partial r}, \quad (14)$$

$$h_{(2)} = -\frac{1}{2} \left[ \left(\frac{\partial \chi_{(1)}}{\partial r}\right)^2 + \left(\frac{\partial \chi_{(1)}}{\partial z}\right)^2 \right]. \quad (15)$$

This way all thermodynamic quantities are expressed using the poloidal-velocity potential.

To progress further, we need a particular form of the perturbation  $\chi_{(1)}(\mathbf{r}, t)$ . This is a difficult global problem that often involves numerical calculations. Several authors studied it under different simplifications. For example, Blaes (1985) gives all possible modes (e.g. eigenfrequencies and eigenfunctions) of slender-torus oscillations. In this limit the size of the stationary torus is small enough that the enthalpy can be approximated by a quadratic function in whole the torus. Blaes (1985) considered the Newtonian gravitational field. Recently, Kluzniak & Abramowicz (2002) reconsidered the problem in general relativity potential and pointed to the existence a particular mode when the torus moves rigidly up and down across the equatorial plane (see also Abramowicz et al. 2005 for more detailed calculations). The eigenfrequency of this mode is equal to the vertical epicyclic frequency  $\omega_z$ . The presence of rigid modes in a torus oscillations has been found also in recent numerical simulations (e.g. Lee et al. 2004; Rubio-Herrera & Lee 2005).

In the following, we model vertical disk oscillations by a simple ansatz for the poloidal-velocity potential

$$\chi_{(1)} = z v_z \cos \omega t, \quad (16)$$

where  $\omega$  is the frequency of the oscillations. Calculating velocity perturbation, we find

$$\mathbf{v}_{(1)} = v_z \mathbf{e}_z \cos \omega t. \quad (17)$$

Hence,  $\epsilon v_z = \text{const}$  can be interpreted as the amplitude of the vertical velocity. Equations (14) and (15) give

$$h_{(1)} = z v_z \omega \sin \omega t, \quad h_{(2)} = -\frac{1}{2} v_z^2 \cos^2 \omega t. \quad (18)$$

The vertical profile of the enthalpy at  $r = r_{\text{in}}$  reads

$$\begin{aligned} h(r_{\text{in}}, z, t) &= h^* - \kappa^2 z^2 + \epsilon z v_z \omega \sin \omega t - \\ &- \frac{1}{2} \epsilon^2 v_z^2 \cos^2 \omega t + \mathcal{O}(\epsilon^3) \end{aligned} \quad (19)$$

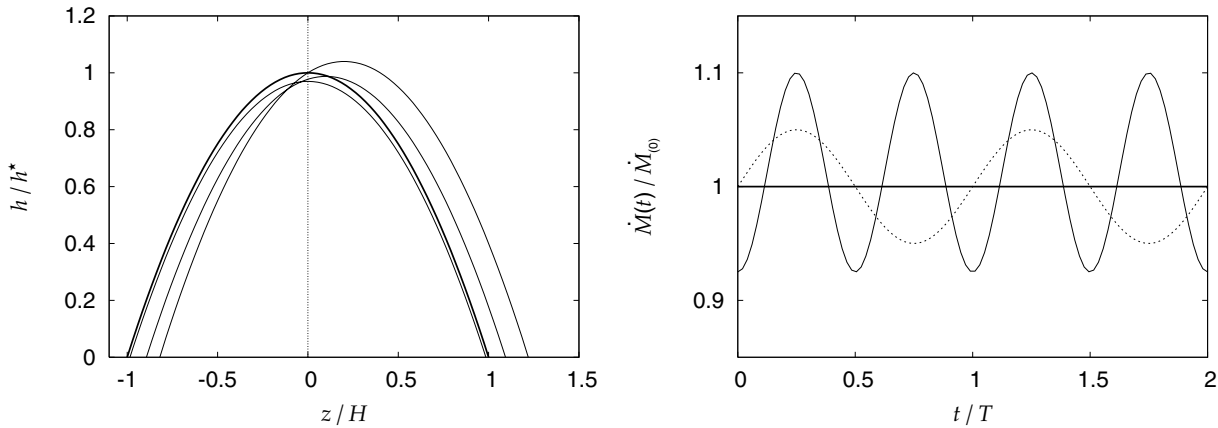
that is quadratic in the variable  $z$ . The position of the enthalpy maximum on the cylinder  $r = r_{\text{in}}$  is shifted from  $z = 0$  to height  $\delta z(t)$  given as

$$\delta z(t) = \delta Z \sin \omega t, \quad \delta Z = \epsilon \frac{\omega v_z}{\kappa^2}. \quad (20)$$

We can interpret  $\delta Z$  as the amplitude of the oscillations. Also the value of enthalpy in the maximum differs from the stationary case by

$$\begin{aligned} \delta h^* &\equiv h(r_{\text{in}}, \delta z) - h^* \\ &= \frac{1}{2} \kappa^2 \left[ \delta z^2 - \frac{\kappa^2}{\omega^2} (\delta Z^2 - \delta z^2) \right] + \mathcal{O}(\epsilon^3). \end{aligned} \quad (21)$$

According to Eq. (6) the actual accretion rate depends on the maximal enthalpy as  $\dot{M} \propto (h^*)^{n+1}$ . This relation can be applied also in the case of vertical oscillations because the



**Fig. 2.** Left: The vertical profiles of the enthalpy  $h(r_{\text{in}}, z)$  on the cylinder  $r = r_{\text{in}}$  during vertical disk oscillations (thin lines). The amplitude of oscillations is  $\delta Z = 0.2H$  and we chose the polytropic index of the fluid  $n = 1.5$ . The figure captures profiles with the enthalpy maxima at  $\delta z = 0, 0.1H$  and  $0.2H$ . The enthalpy profile for the unperturbed stationary disk is also shown (thick line). Right: The modulated accretion rate from the oscillating disk (thin solid line). The accretion rate for the stationary disk is plotted by thick line. Time is rescaled by the period of oscillations. For reference we plot also the phase of disk oscillations (dotted line). The accretion rate is modulated with twice the frequency of oscillations.

$z$ -dependence of enthalpy on the cylinder  $r = r_{\text{in}}$  remains quadratic also in this case and the oscillations do not contribute to the radial velocity of accreted matter. Hence, using Eqs. (4), (8) and (21) and assuming that the frequency of oscillations equals to the local vertical epicyclic frequency,  $\omega = \omega_z$ , we arrive at our final result

$$\begin{aligned} \frac{\delta \dot{M}}{\dot{M}_{(0)}} &= (n+1) \frac{\delta h^*}{h^*} \\ &= \frac{2-p}{2-2p} \left[ (1+p) \frac{\delta z^2}{H^2} - p \frac{\delta Z^2}{H^2} \right], \end{aligned} \quad (22)$$

where  $\delta M \equiv \dot{M} - \dot{M}_{(0)}$  and  $p \equiv n/(n+1/2)$ .

Figure 2 shows the result. The enthalpy profiles  $h(r_{\text{in}}, z)$  are shown for several values of  $\delta z$  in the left panel. The amplitude of oscillations is  $\delta Z/H = 0.3$ . The right panel shows the modulation of the accretion rate from the oscillating disk. The time is rescaled by the oscillation period,  $T = 2\pi/\omega_z$ . Finally, the time-averaged accretion rate is given by

$$\frac{\langle \delta \dot{M} \rangle}{\dot{M}_{(0)}} = \frac{1}{4}(2-p) \frac{\delta Z^2}{H^2} \quad (23)$$

that is positive for reasonable values of  $n$ .

## 5. Conclusions

In this note we studied the accretion rate from a non-stationary pressure supported accretion disk that undergoes the vertical axisymmetric oscillations. The oscillations were modelled by a simple ansatz for the perturbation of poloidal-velocity field. We believe, however, that several features would be present also in more sophisticated (perhaps numerical) solutions: (1) the first correction to the stationary accretion rate is of the quadratic order in both the actual perturbation  $\delta z$  and the amplitude  $\delta Z$ . This is probably because of the symmetry of the stationary flow with respect to the equatorial plane. Hence, the frequency of the modulation must be twice the oscillation frequency. (2) The accretion rate is maximal

when the disk reaches the maximal amplitude  $\delta z = \delta Z$ . (3) The averaged accretion rate from the periodically perturbed flow is greater than the that of the stationary flow.

*Acknowledgements.* The author is grateful to Marek Abramowicz for the invitation to participate in *Nordita days on QPOs* conference and also to Wlodek Kluźniak and Vladimír Karas for helpful discussion about the subject. This work was supported by the GA AV grant IAA 300030510.

## References

- Abramowicz, M.A., Jaroszyński, M., Sikora, M.: 1978, *A&A* 63, 221
- Abramowicz, M.A.: 1985, *PASJ* 37, 727
- Abramowicz, M.A., Kluźniak, W.: 2001, *A&A* 374, L19
- Abramowicz, M.A., Blaes, O.M., Horák, J., Kluźniak, W., Rebusco, P.: 2005, in preparation
- Blaes, O.M.: 1985, *MNRAS* 216, 553
- Boyer, R.H.: 1965, *Proc. Cambridge Phil. Soc.* 61, 527
- Bursa, M., Abramowicz, M.A., Karas, V., Kluźniak, W.: 2004, *ApJ* 617, L45
- Gilfanov, M., Revnivtsev, M., Molkov, S.: 2003, *A&A* 410, 217
- Kluźniak, W., Michelson, P., Wagoner, R.V.: 1990, *ApJ* 358, 538
- Kluźniak, W., Abramowicz, M.A.: 2001, *AcPPB* 32, 3605
- Kluźniak, W., Abramowicz, M.A.: 2002, *astro-ph/0203314*
- Kluźniak, W., Abramowicz, M.A.: 2004, *Amsterdam proceedings*, in press
- Kozłowski, M., Jaroszyński, M., Abramowicz, M.A.: 1978, *A&A* 63, 209
- Lee, W.H., Abramowicz, M.A., Kluźniak, W.: 2004, *ApJ* 603, L93
- McClintock, J.E., Remillard, R.A.: 2003, in: W.H.G. Lewin, M. van der Klis (eds.), *Compact Stellar X-ray Sources*, Cambridge Univ. Press (astro-ph/0306213)
- Paczynski, B.: 1987, *Nature* 327, 303
- Paczynski, B., Wiita, P.J.: 1980, *A&A* 88, 23
- Revnivtsev, M., Gilfanov, M.: 2005, *astro-ph/0506019*
- Rubio-Herrera, E., Lee, W.H.: 2005, *MNRAS* 357, L31
- Sunyaev, R.A., Shakura, N.I.: 1986, *SvAL* 12, 117
- van der Klis, M.: 2000, *ARA&A* 38, 717