Amplitude behaviour in HFQPOs in neutron stars

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in collaboration with

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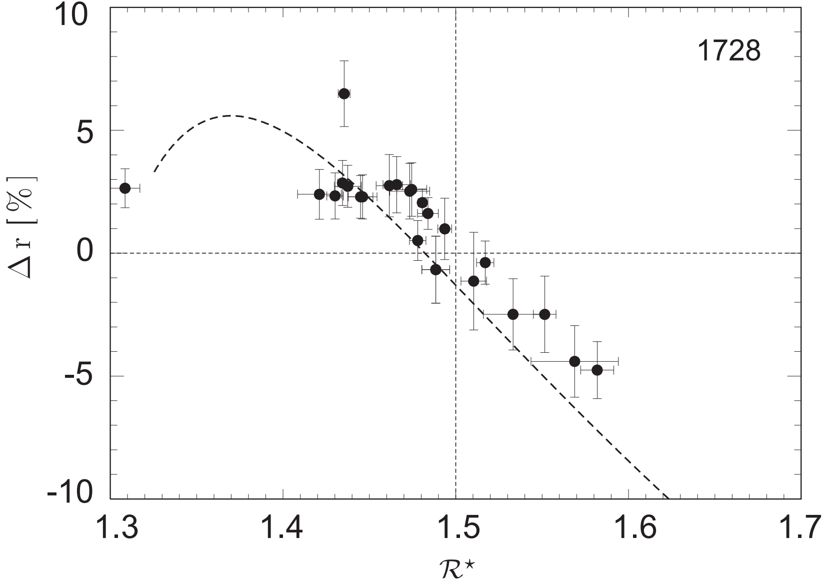
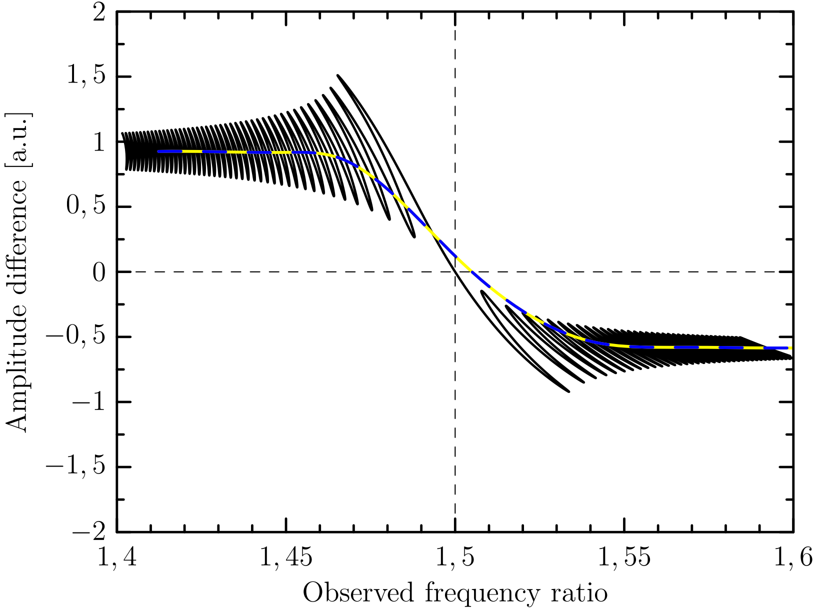
Motivation

Amplitude behaviour of neutron-star QPOs



... Török (2008?)

Passage through the resonance?



What's going on?

Coupled two oscillations

The governing equations

$$\begin{aligned} \ddot{x}_\ell + \omega_\ell^2 x_\ell &= f_\ell(x_\ell, x_u, \dot{x}_\ell, \dot{x}_u) \\ \ddot{x}_u + \omega_u^2 x_u &= f_u(x_\ell, x_u, \dot{x}_\ell, \dot{x}_u) \end{aligned}$$

Conservation of the energy

$$f(x_\ell, x_u, -\dot{x}_\ell, -\dot{x}_u) = f(x_\ell, x_u, \dot{x}_\ell, \dot{x}_u)$$

Small oscillations (weak coupling): The solutions have the form

$$x_\ell(t) = a_\ell(t)\mathrm{e}^{\mathrm{i}[\omega_\ell t + \phi_\ell(t)]}, \quad x_u(t) = a_u(t)\mathrm{e}^{\mathrm{i}[\omega_u t + \phi_u(t)]},$$

Resonant behavior (I)

- The amplitudes a_ℓ and a_u are governed by

$$\boxed{\frac{\mathrm{d}a_\ell}{\mathrm{d}t} = \tfrac{1}{16}\beta\nu a_\ell^2 a_u^2 \sin\gamma, \quad \frac{\mathrm{d}a_u}{\mathrm{d}t} = -\tfrac{1}{16}\beta a_\ell^3 a_u \sin\gamma}$$

and the phase function $\gamma \equiv 2(\phi_u + \omega_u t) - 3(\phi_\ell + \omega_\ell t)$

$$\boxed{\frac{\mathrm{d}\gamma}{\mathrm{d}t} = 2\omega_u - 3\omega_\ell + \tfrac{1}{4}\left[\mu_\ell a_\ell^2 + \mu_u a_u^2 + \tfrac{1}{2}\beta a_\ell\left(-a_\ell^2 + \tfrac{3}{2}\nu a_u^2\right)\cos\gamma\right]}$$

- Eigenfrequency vs. observed frequency ratio

$$\mathcal{R} = \frac{\omega_u}{\omega_\ell}, \qquad \mathcal{R}^* = \frac{3}{2}\left(1 + \frac{\dot\gamma}{2\omega_u}\right)$$

- $\dot\gamma = 0 \Rightarrow$ Observed frequencies in 3:2 ratio.

Resonant behavior (II)

Conservation of the energy: $a_\ell^2 + \nu a_u^2 = E$

$$a_\ell^2 = \xi(t)E, \quad a_u^2 = \frac{1}{\nu}[1 - \xi(t)]E$$

The evolution of the system is by

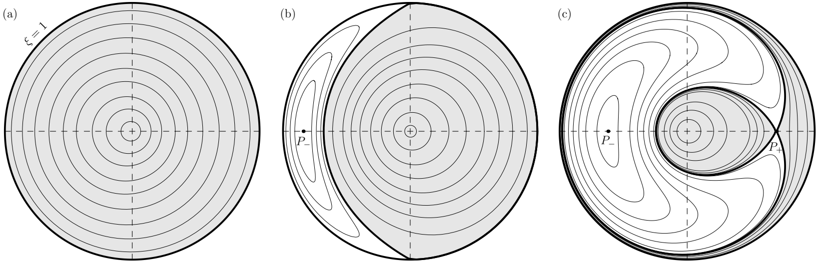
$$\boxed{\dot{\xi} = \tfrac{1}{8}\beta(E\xi)^{3/2}(1-\xi)\sin\gamma}$$

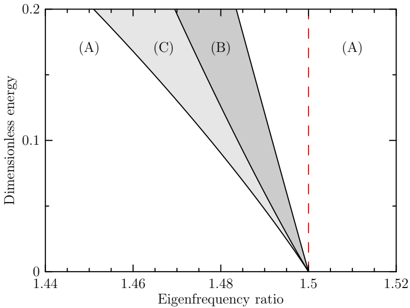
and

$$\boxed{\dot{\gamma} = 2\omega_u - 3\omega_\ell + \tfrac{1}{4}E\left[\mu_\ell\xi + \frac{\mu_u}{\nu}(1-\xi) + \tfrac{1}{4}\beta(E\xi)^{1/2}(3-5\xi)\cos\gamma\right]}$$

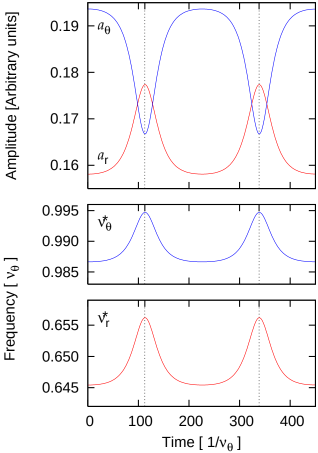
Critical points: $\dot{\gamma} \equiv \dot{\xi} \equiv 0$

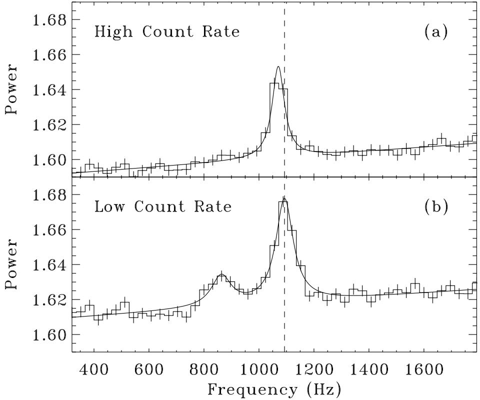
Resonance tongues and topologies





Remark: Sco X-1 amplitude and frequencies





Low-frequency modulation ($\nu_{\rm NBO} = 6$ Hz) of HF QPOs

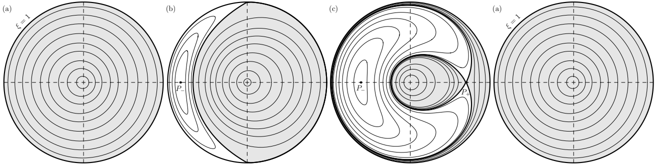
Effects of slow change of eigenfrequencies

e.g. *Motion or expansion of the accretion torus, ...*

$$\mathcal{R} = \mathcal{R}(t), \quad t_{\mathrm{char}} \ll \frac{1}{\omega_u}$$

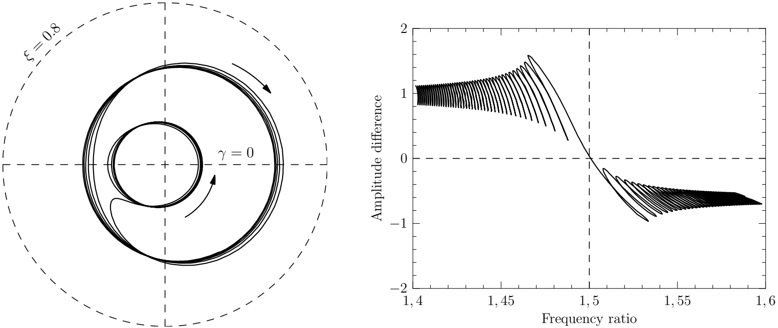
The position in the $(\mathcal{R}, E)$ plane is slowly changed

$\Rightarrow$ Change of the topology:



Few results

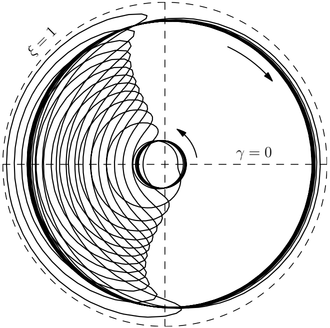
1. Redistribution of the energy

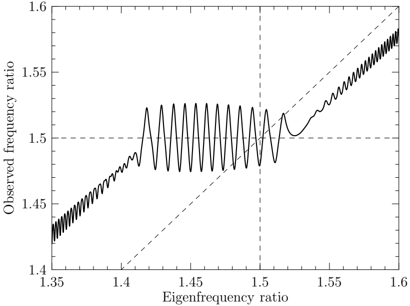


Parameteres of the solution:

$\beta = -\mu_\ell = \mu_u = 5$, $\nu = \frac{9}{4}$, $E = 0.1$, $\mathcal{R} = 1.425 - 1.650$,
$T = 10^4 \omega_u$.

2. "Phase-locking" period

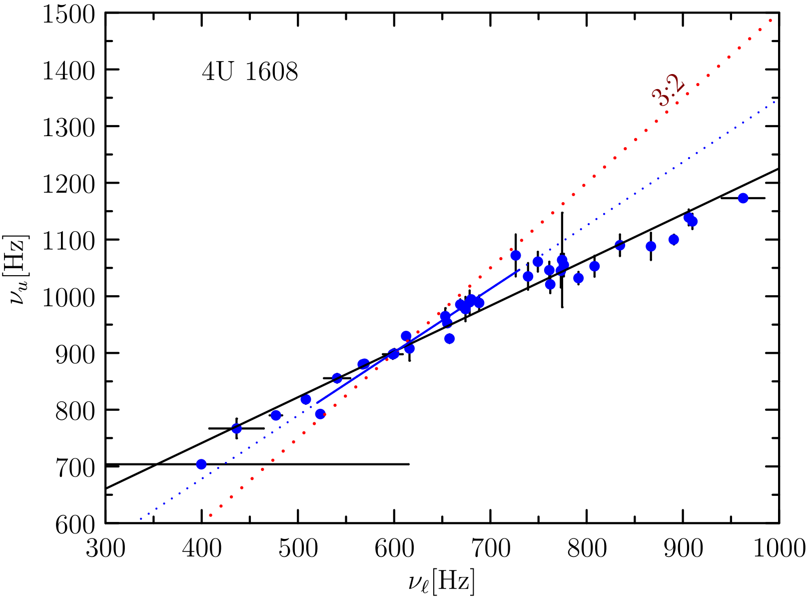




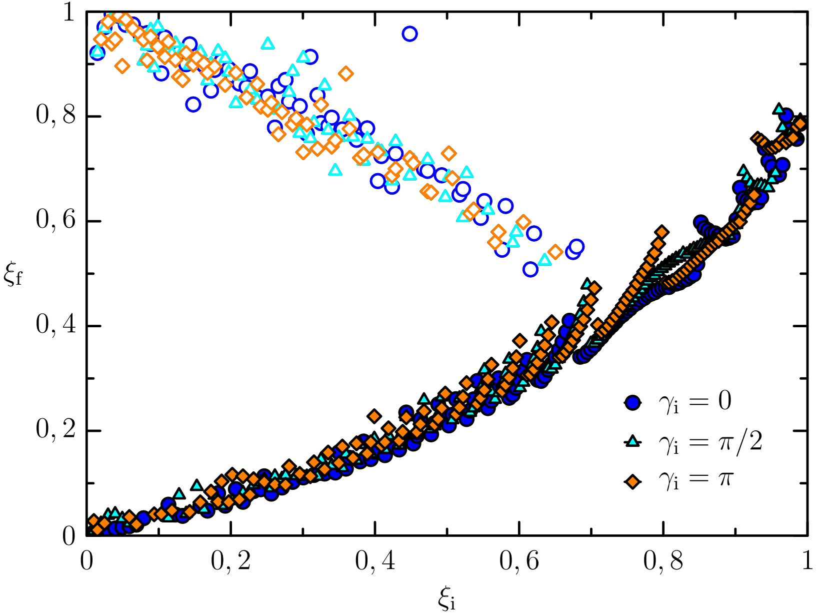
Parameteres of the solution:

$\beta = \mu_\ell = -\mu_u = 5,\ \nu = \frac{9}{4},\ \mathcal{R} = 1.35 - 1.65,\ T = 10^4\omega_u.$

Remark: Could this be observed?



1. Dependence on the initial condition



Etc...

Discussion and conclusions – I.

- Jump in the amplitudes as the system pass the resonance tongues in the $(\mathcal{R}, E)$ plane.
- The observed frequencies are in *exact* 3:2 ratio.

However (TBA)

- RMS $\rightarrow a_\ell$, a_u? (modulation mechanism?)
- Non-conservative systems (limit cycles, ...)
- Specific models?