Characteristic sound of black-hole disk accretion

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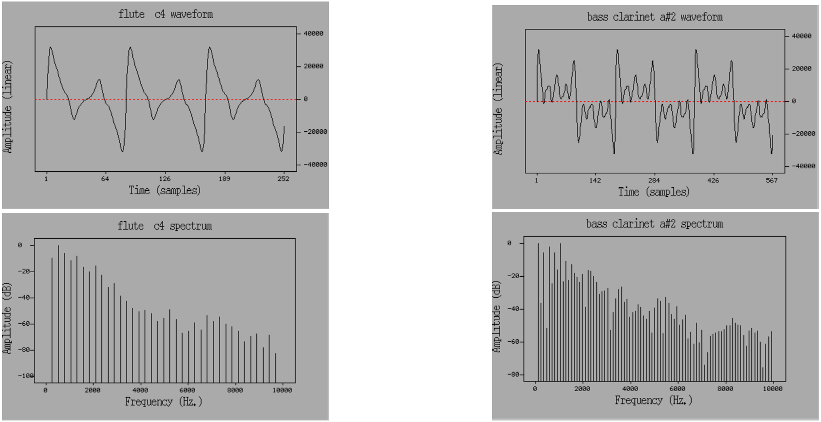
17.9.2010, Opava

Prologue

Everything has its sound...



What is the characteristic sound of accretion disks?

Outline

- *Physics* of disk oscillations in GR
  $\rightarrow$ Governing equations in GR
  $\rightarrow$ Master equation for *p*-modes
  $\rightarrow$ Singular terms, boundary conditions
- QNM, Characteristic sound of accretion disks
  $\rightarrow$ Preliminary results on lowest-order modes
- Results of numerical *simulations*
  $\rightarrow$ 2d, 3d, viscous, turbulent
- *Observations* of real systems
  $\rightarrow$ QPOs? which one?, GRS 1915+105
- Discussion and conclusions

Disk oscillations in GR

Stationary accretion disk model

- Stationary metric

$$\boxed{ds^2 = g_{tt}dt^2 + 2g_{t\phi}dtd\phi + g_{\phi\phi}d\phi^2 + g_{rr}dr^2 + g_{zz}dz^2}$$

Inverse metric tensor components ($\mathcal{R} = \sqrt{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}$)

$$g^{tt} = -g_{\phi\phi}/\mathcal{R}^2, \quad g^{t\phi} = g_{t\phi}/\mathcal{R}^2, \quad g^{\phi\phi} = -g_{tt}/\mathcal{R}^2, \quad g^{ii} = 1/g_{ii},$$

- Purely azimuthal motion of fluid in equatorial plane

$$u^\alpha = u^t(\delta^\alpha_t + \Omega\delta^\alpha_\phi), \quad u_\beta = u_t(\delta^t_\beta - \ell\delta^\phi_\beta).$$

Angular momentum and agular velocity

$$\Omega = \frac{g^{t\phi} - \ell g^{\phi\phi}}{g^{tt} - \ell g^{t\phi}}, \quad \ell = -\frac{g_{t\phi} + \Omega g_{\phi\phi}}{g_{tt} + \Omega g_{t\phi}} \equiv \ell_{\mathrm{K}}$$

- Polytropic fluid, $p \propto \rho^{1+1/n}$, no viscosity $\alpha = 0$.

Dynamics

- Ipser & Lindblom (1991) formalism or direct calculations

- Euler equation

$$\nabla_\alpha T^\alpha_\beta = 0, \quad T^\alpha_\beta = (e + p) u^\alpha u_\beta + p \delta^\alpha_\beta,$$

- Continuity equation

$$\nabla_\alpha \left(\rho u^\alpha\right) = \frac{1}{\sqrt{-g}} \partial_\alpha \left(\sqrt{-g} \rho u^\alpha\right) = 0.$$

- Axial symmetry:

$$\delta \propto \exp\left[-\mathrm{i}(\omega t - m\phi\right], \quad u^\alpha \partial_\alpha = -\mathrm{i} u^t (\omega - m\Omega) = -\mathrm{i} u^t \sigma$$

Perturbations – I.

$$(\ldots)u_\beta + a_\beta(\delta e + \delta p) + (e+p)\,\delta a_\beta + \nabla_\beta\,\delta p = 0,$$

- Four-acceleration of the flow: $a_\beta = u^\alpha \nabla_\alpha u_\beta$

$$\delta a_\beta = u^\alpha \delta u_{\beta,\alpha} + u_{\beta,\alpha}\delta u^\alpha - g_{\mu\nu,\beta} u^\mu \delta u^\nu$$

- Normalization of the four-velocity: $\delta u_t = -\Omega \delta u_\phi$, $\delta u^t = \ell \delta u^\phi$.

- Contraction with $w^\beta = \delta^\beta_\phi + \ell \delta^\beta_t$:

$$\boxed{(e+p)\left[\mathrm{i}\sigma u^t \frac{\delta u_\phi}{u_t} - u_t \ell_{,k} \delta u^k\right] - \mathrm{i}(\ell\omega - m)\delta p = 0}$$

- Poloidal components ($i = r, z$):

$$\boxed{(\delta e + \delta p)a_i + (e+p)\left[i\sigma u^t \delta u_i - \mathcal{A}_i \frac{\delta u_\phi}{u_t}\right] + \delta p_{,i} = 0}$$

Perturbations – II.

Continuity equation

$$\frac{1}{\sqrt{-g}}\partial_k\left(\sqrt{-g}\rho\,\delta u^k\right)+\mathrm{i}(\ell\omega-m)\rho\frac{1}{\mathcal{R}^2}\frac{u_t^2}{u^t}\frac{\delta u_\phi}{u_t}-\mathrm{i}\sigma u^t\delta\rho=0$$

Vertical integration (hopefully fine for *p*-modes):

$$\begin{aligned}
-\mathrm{i}\sigma\frac{\delta u_\phi}{u_t}+u_t\ell_{,r}\delta u^r+\mathrm{i}(\ell\omega-m)\delta h &= 0\\
-\mathrm{i}\sigma u^t\delta u_r-\mathcal{A}\frac{\delta u_\phi}{u_t}+\delta h_{,r} &= 0\\
-\mathrm{i}\sigma u^t\frac{\Sigma}{c_s^2}\delta h+\frac{1}{\sqrt{-g}}\partial_r\left(\sqrt{-g}\Sigma\,\delta u^r\right)+\mathrm{i}\Sigma(\ell\omega-m)\frac{1}{\mathcal{R}^2}\frac{u_t^2}{u^t}\frac{\delta u_\phi}{u_t} &= 0
\end{aligned}$$

where

$$dh=\frac{dp}{\rho},\quad \mathcal{A}=\frac{\Omega_{,r}}{1-\ell\Omega}-\frac{u_t^3}{u^t}\frac{\ell_{,r}}{\mathcal{R}^2},\quad \kappa^2=\frac{1}{g_{rr}}\frac{u_t}{u^t}\mathcal{A}.$$

Master equation – I

Radial and “azimuthal” Euler equation ($D = \kappa^2 - \sigma^2$):

$$\begin{aligned} \delta u^r &= \frac{\mathrm{i}}{g_{rr} u^t D} \left[\sigma \delta h_{,r} - (\ell\omega - m)\mathcal{A}\,\delta h\right] \\ \frac{\delta u_\phi}{u_t} &= \frac{1}{u^t D} \left[g_{rr} \ell_{,r} u_t\, \delta h_{,r} - \sigma(\ell\omega - m) u^t\, \delta h\right] \end{aligned}$$

Substitution into continuity equation + some algebra

$$\begin{aligned} \delta h_{,rr} &- \partial_r \ln\left(\frac{g_{rr} D (u^t)^2}{\sqrt{-g}\Sigma}\right) \delta h_{,r} - \\ &- \left\{\frac{\ell\omega - m}{\sigma} \mathcal{A} \partial_r \ln\left(\frac{(\ell\omega - m)\sqrt{-g}\Sigma\mathcal{A}}{g_{rr} u^t D}\right) - \right. \\ &- \left. g_{rr}(u^t)^2 \left[\frac{1}{\mathcal{R}^2}(\ell\omega - m)^2 + \frac{D}{c_s^2}\right]\right\} \delta h = 0. \end{aligned}$$

Master equation – II

Introduce

$$x = \int_0^r g_{rr}^{1/2} u^t \mathrm{d}r, \quad \eta = S^{-1/2}\delta h, \quad S = u^t \left(\frac{g_{rr}}{-g}\right)^{1/2} \frac{D}{\Sigma}$$

and obtain Schrödinger equation

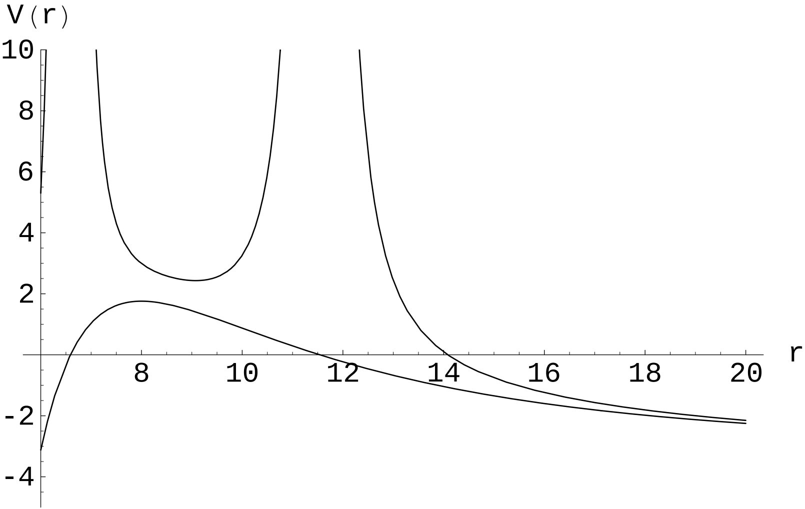
$$\boxed{\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2} - \mathcal{V}(\sigma, x)\right]\eta = 0}$$

with the “potential” (only dominant terms when $c_s \to 0$)

$$\mathcal{V} = -k^2 + \frac{2}{k^2}\left(\frac{\mathrm{d}k}{\mathrm{d}x}\right)^2 - \frac{1}{k}\frac{\mathrm{d}^2 k}{\mathrm{d}x^2} + \frac{(\ell\omega - m)\mathcal{A}}{g_{rr}(u^t)^2\sigma}\partial_r \ln\left[\frac{(\ell\omega - m)\sqrt{-g}\Sigma\mathcal{A}}{g_{rr}u^t D}\right]$$

- $k^2 = -\sqrt{\sigma^2 - \kappa^2}/c_s$, $k \to 0$ at Lindblad resonances
- $\sigma = \omega - m\Omega$, $\sigma \to 0$ at the corotation radius

Axisymmetric modes



Lowest-order axisymmetric *p*-modes

Boundary conditions

WKBJ approximation

$$\eta = Ak^{-1/2} \exp\left(\mathrm{i} \int^r k \,\mathrm{d}x\right) + Bk^{-1/2} \exp\left(-\mathrm{i} \int^r k \,\mathrm{d}x\right)$$

Λ ratio of the ingoing and outgoing wave amplitudes, then

$$2(1+\Lambda)k\eta' + \left[(1+\Lambda)k' + 2\mathrm{i}(1-\Lambda)k^{5/2}\right]\eta = 0$$

- outer boundary: outgoing wave only $\Rightarrow \Lambda = \infty$

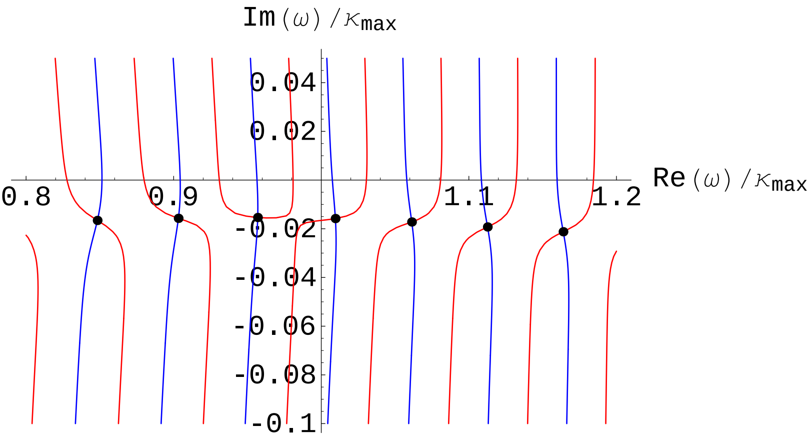
$$\boxed{2k\eta' + (k' - 2\mathrm{i}k^{5/2})\eta = 0}$$

- inner boundary: ???, worst case no reflection $\Rightarrow \Lambda = 0$

$$\boxed{2k\eta' + (k' + 2\mathrm{i}k^{5/2})\eta = 0}$$

Lowest-order modes – I.

Runge-Kutta, shooting method, *etc*...



Def. $\Delta = \eta_{\mathrm{R}}(r_0)\eta'_{\mathrm{L}}(r_0) - \eta_{\mathrm{L}}(r_0)\eta'_{\mathrm{R}}(r_0)$. Eigenmodes $\Leftrightarrow$ $\boxed{\Delta = 0}$.

Results

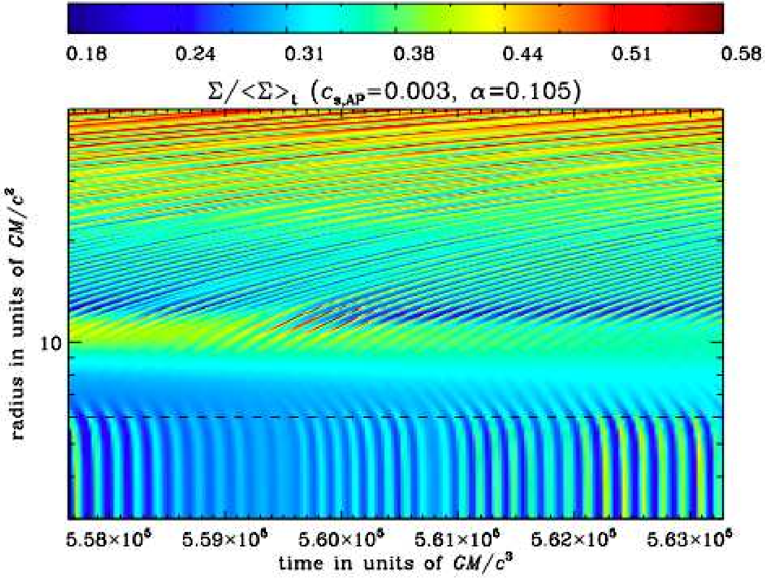
- Lowest-order modes for $\Lambda_{\text{in}} = 0$ (no reflection at ISCO)

$\text{Re}(\omega)/\kappa_{\text{max}}$	$\text{Im}(\omega)/\kappa_{\text{max}}$	Q
0.847	0.0161	52.4
0.900	0.0136	65.9
0.936	0.0067	138
0.969	0.0122	79.2
1.020	0.0135	75.5

- Lowest damping for modes with $\omega \approx \kappa_{\text{max}}$
- Dominant in the late-time response.
- Analogic to the black-hole quasi-normal modes.

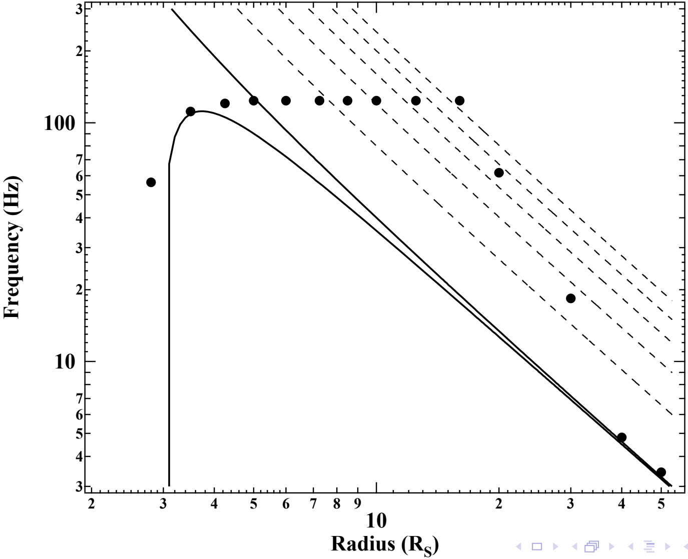
Simulations

Vertically integrated α-disk (Barbara Fereira, 2010)



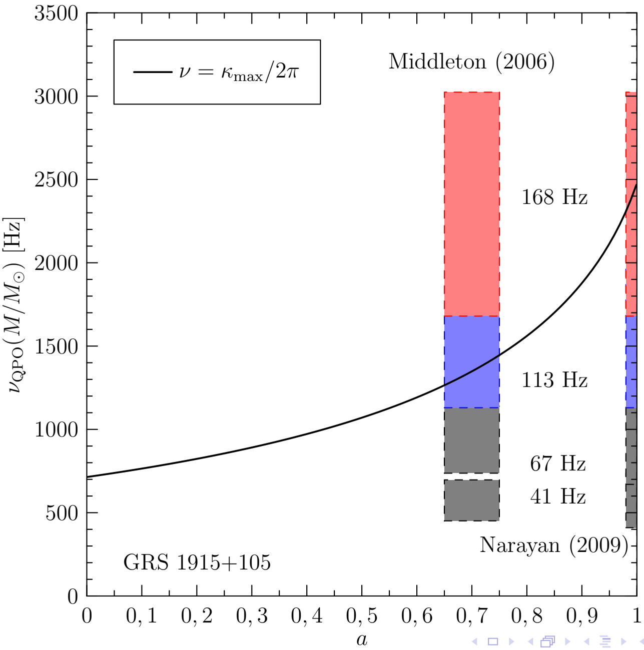
Vertically integrated α-disk (Mao, 2009)

Dominant frequency in the power spectra.



QPOs

HF QPOs in GRS 1915+105



Conclusions

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- Quasi-normal modes dominate the late-time response
- Consequence of GR: maximum of $\kappa \rightarrow$ reproducibility, $1/M$-scaling, *etc*...$\Rightarrow$ QPOs? $\rightarrow$ spin determination
- Calculation of the response to arbitrary initial perturbation? $\Rightarrow$ QNM dominant in the late-time phase
- Effect of viscosity/turbulence in excitation/damping?