

Corotation resonance in black-hole accretion disks

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Stationary accretion disk model

- ▶ Stationary metric

$$ds^2 = g_{tt} dt^2 + 2g_{t\phi} dt d\phi + g_{\phi\phi} d\phi^2 + g_{rr} dr^2 + g_{zz} dz^2$$

Inverse metric tensor components ($\mathcal{R} = \sqrt{g_{t\phi}^2 - g_{tt} g_{\phi\phi}}$)

$$g^{tt} = -g_{\phi\phi}/\mathcal{R}^2, \quad g^{t\phi} = g_{t\phi}/\mathcal{R}^2, \quad g^{\phi\phi} = -g_{tt}/\mathcal{R}^2, \quad g^{ij} = 1/g_{ii},$$

- ▶ Purely azimuthal motion of fluid in equatorial plane

$$u^\alpha = u^t(\delta_t^\alpha + \Omega \delta_\phi^\alpha), \quad u_\beta = u_t(\delta_\beta^t - \ell \delta_\beta^\phi).$$

- ▶ Thin disk: $p, e \ll \rho \rightarrow$ pressure gradients dynamically unimportant
- ▶ Keplerian angular momentum distribution: $\ell = \ell_K$
- ▶ Polytropic fluid, $p \propto \rho^{1+1/n}$
- ▶ No viscosity, no turbulence, $\alpha = 0$.

Dynamics

- ▶ Ipser & Lindblom (1991) formalism or direct calculations
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- ▶ Euler equation

$$\nabla_{\alpha} T_{\beta}^{\alpha} = 0, \quad T_{\beta}^{\alpha} = (e + p)u^{\alpha}u_{\beta} + p\delta_{\beta}^{\alpha},$$

- ▶ Continuity equation

$$\nabla_{\alpha} (\rho u^{\alpha}) = \frac{1}{\sqrt{-g}} \partial_{\alpha} (\sqrt{-g} \rho u^{\alpha}) = 0.$$

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- ▶ Axial symmetry:

$$\delta \propto \exp[-i(\omega t - m\phi)], \quad \tilde{\omega} = \omega - m\Omega, \quad \tilde{m} = m - \ell\omega$$

Perturbations

Vertical integration (hopefully fine for p -modes):

$$-i\tilde{\omega} \frac{\delta u_\phi}{u_t} + u_t \ell_{,r} \delta u^r - i\tilde{m} \delta h = 0,$$

$$-i\tilde{\omega} u^t \delta u_r - \mathcal{A} \frac{\delta u_\phi}{u_t} + \delta h_{,r} = 0,$$

$$-i\tilde{\omega} u^t \frac{\Sigma}{\tilde{c}_s^2} \delta h + \frac{1}{\sqrt{-g_3}} \partial_r \left(\sqrt{-g_3} \Sigma \delta u^r \right) - i\Sigma \tilde{m} \frac{1}{\mathcal{R}^2} \frac{u_t^2}{u^t} \frac{\delta u_\phi}{u_t} = 0.$$

where

$$dh = \frac{dp}{\rho}, \quad \mathcal{A} = \frac{\Omega_{,r}}{1 - \ell\Omega} - \frac{u_t^3}{u^t} \frac{\ell_{,r}}{\mathcal{R}^2}, \quad \kappa^2 = \frac{1}{g_{rr}} \frac{u_t}{u^t} \mathcal{A}.$$

Master equation

Introducing

$$\eta = S^{-1/2} \delta h, \quad S = u^t \left(\frac{g_{rr}}{-g} \right)^{1/2} \frac{\kappa^2 - \tilde{\omega}^2}{\Sigma}$$

one obtains Schrödinger equation

$$\left[\frac{d^2}{dr^2} - V_{\text{eff}}(\omega, r) \right] \eta(r) = 0$$

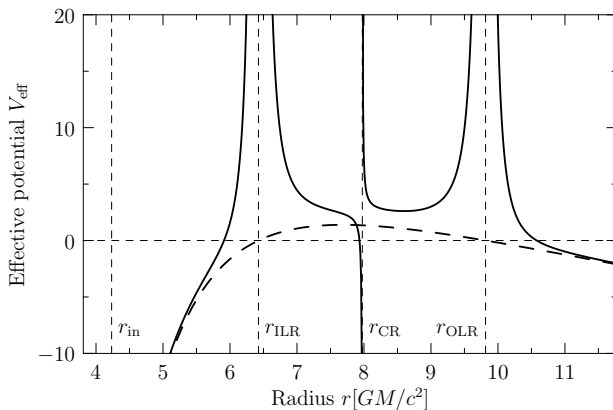
with the “potential” (only dominant terms when $c_s \rightarrow 0$)

$$V_{\text{eff}}(\omega, r) = -k^2 + k \partial_r^2 \left(\frac{1}{k} \right) - \frac{\tilde{m} \mathcal{A}}{\tilde{\omega}} \partial_r \ln \left(\frac{\sqrt{-g_3}}{g_{rr}} \frac{\tilde{m} \mathcal{A} \Sigma}{Du^t} \right),$$

- ▶ $k^2 = g_{rr}^{1/2} u^t \sqrt{\tilde{\omega}^2 - \kappa^2} / \bar{c}_s$ dominant term (WKBJ)
- ▶ $k \rightarrow 0$ at Lindblad resonances
- ▶ $\tilde{\omega} \rightarrow 0$ at the corotation resonance

Propagation diagram

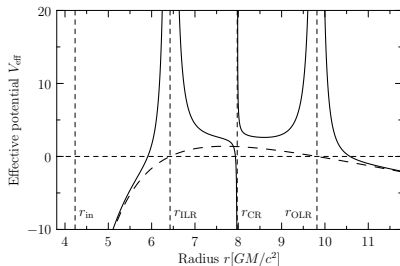
$$V_{\text{eff}}(\omega, r) = -k^2 + k \partial_r^2 \left(\frac{1}{k} \right) - \frac{\tilde{m}\mathcal{A}}{\tilde{\omega}} \partial_r \ln \left(\frac{\sqrt{-g_3} \tilde{m}\mathcal{A}\Sigma}{g_{rr} Du^t} \right),$$



- ▶ $V_{\text{eff}} < 0$: wave-propagation region, $V_{\text{eff}} > 0$: evanescent region
- ▶ $r < r_{\text{CR}}$: positive-energy waves (*increasing* the energy of the flow)
- ▶ $r > r_{\text{CR}}$: negative-energy waves (*decreasing* the energy of the flow)

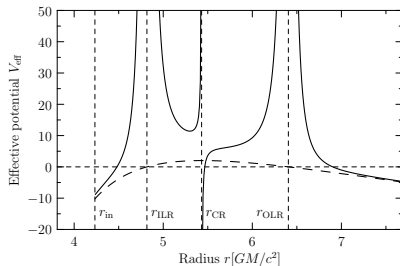
Corotation resonance

- ▶ $r = r_{\text{CR}}$: corotation resonance \Rightarrow *absorption of the waves*



- ▶ negative-energy waves absorbed

DAMPING



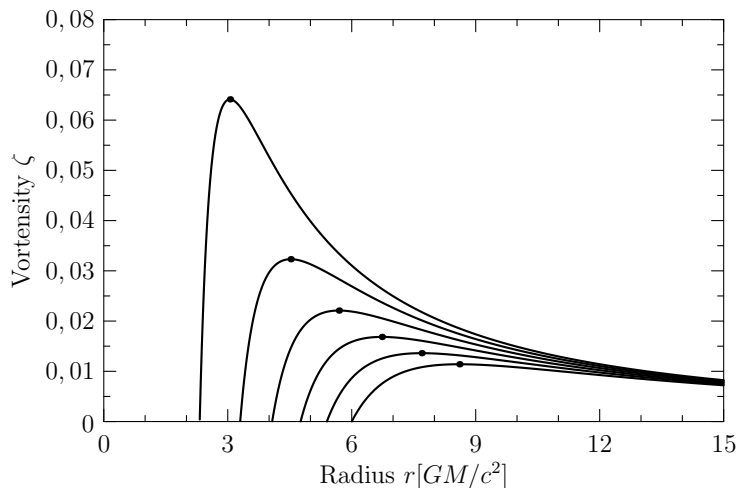
- ▶ positive-energy waves absorbed

INSTABILITY

Condition for the instability in terms of the **relativistic vorticity**

$$\boxed{\left(\frac{d}{dr} \ln \zeta\right)_{r=r_{\text{CR}}} > 0} \quad \zeta = \frac{g_{rr} u^t \kappa^2}{\sqrt{-g_3} \mathcal{A} \Sigma (1 - \ell \Omega)}$$

Relativistic vortensity



... Increasing vortensity with radius only in *relativistic* domain

Results in Kerr spacetime

Disk model

Additional assumptions:

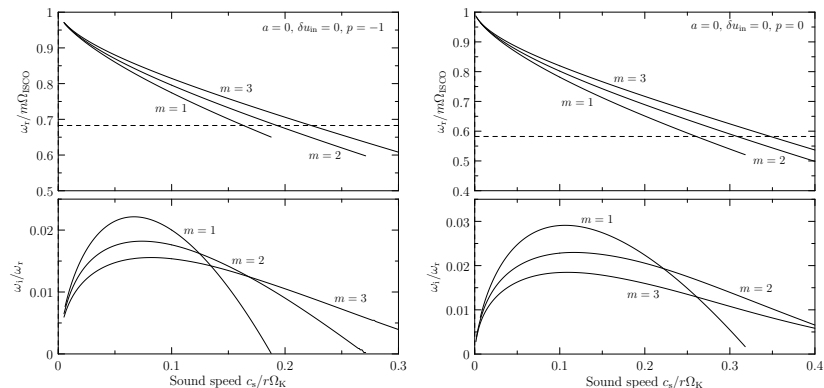
- ▶ Power-law density distribution: $\Sigma \propto r^{-p}$
- ▶ Speed of sound: $\bar{c}_s / (r\Omega_K) = \text{const}$
- ▶ Disk inner edge: $r_{\text{in}} = r_{\text{ISCO}}$

Boundary conditions:

- ▶ Inner boundary: $\delta u(r_{\text{in}}) = 0$
- ▶ Outer boundary: outgoing wave $\rightarrow (d/dr - ik)\delta h = 0$

Results

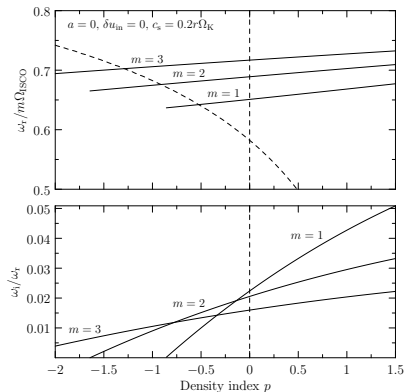
Dependence of the eigenfrequencies on the sound speed



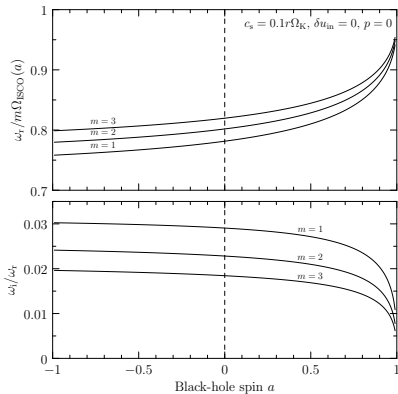
⇒ Nearly harmonic sequence of frequencies: 1:2:3...

Results

Density profile



Black-hole spin



Model for HF QPOs?

